Permeability of porous materials determined from the Euler characteristic

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Motivation

Soil  
Oil Reservoir  
Foams  
Tissue

\[ Q = -k \eta \nabla P, \]  
flow rate  
pressure gradient  
viscosity  
permeability

Which structural information determines \( k \)?
Darcy’s law

\[ Q = -\frac{k}{\eta} \nabla P, \]

flow rate \( Q \), pressure gradient \( \nabla P \), viscosity \( \eta \), permeability \( k \)
Motivation

Soil, Oil Reservoir, Foams, Tissue

Darcy’s law

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flow rate \( Q \), pressure gradient \( \nabla P \), viscosity \( \eta \), permeability \( k \)

Which structural information determines \( k \)?
Katz-Thompson law  

\[ k = c l_c^2 \left( \frac{\sigma}{\sigma_0} \right), \]

critical pore diameter \( l_c \), conductivity \( \sigma \), fluid conductivity \( \sigma_0 \)
Katz-Thompson law  

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Archie’s law  
Archie, Petroleum Transactions of AIME 146: 54–62 (1942)

\[ \frac{\sigma}{\sigma_0} = \left( \frac{\phi - \phi_c}{1 - \phi_c} \right)^\nu, \]

porosity \( \phi \), percolation threshold \( \phi_c \), \( \nu = 1.3 \)
Limitations of Archie’s law

- Archie’s law only valid close to $\phi_c$
- $\phi_c$ cannot be determined from a single sample
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Alternative relation independent of $\phi_c$?
⇒
Minkowski functionals
Minkowski functionals

- Area
- Boundary
- Euler Characteristic
Minkowski functionals


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Minkowski functionals


- Area
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Boolean Models - Randomly overlapping grains

Circles

Ellipses

Porosities $\phi \in [\phi_c, 1]$. 
Microfluidic (Quasi 2D) measurement + LB Simulation

- PDMS structure
- $L$
- $w$
- $h$
- $\Delta P$

Colloidal tracers + video microscopy
Microfluidic (Quasi 2D) measurement + LB Simulation

Colloidal tracers + video microscopy
Results


Circles (▼, ▶), Ellipses (▲, △)
\[ \frac{k}{c l^2} \sim \left( \frac{\phi - \phi_c}{1 - \phi_c} \right) \]

Circles (▼, ▽), Ellipses (▲, △)
Relation to Minkowski functionals

Permeability vs. Euler characteristic

\[ k = c l_c^2 \left( \frac{1 - \chi_o}{N} \right)^\alpha, \]

Euler characteristic of the conducting phase \( \chi_o \), total number of grains \( N \)
Permeability vs. Euler characteristic

\[ k = c l_c^2 \left( \frac{1 - \chi_o}{N} \right)^\alpha, \]

Euler characteristic of the conducting phase \( \chi_o \), total number of grains \( N \)

\[ \rightarrow \text{Independent of } \phi_c \]

$\frac{k}{c l_c^2} = 10 \times (1 - \chi_o)/N$

Circles (▼, ▽), Ellipses (▲, △)
Relation to $\chi_0$


$\frac{k}{cl^2}$ vs. $\phi_0$

Circles (▼, ▽), Ellipses (▲, △)
Velocity fields

\[ \frac{1 - \chi_0}{N} \]
Conclusion

- Permeability determined from Euler Characteristic independent of $\phi_c$
- Applicability to arbitrary structures?
Conclusion

- Permeability determined from Euler Characteristic independent of $\phi_c$
- Applicability to arbitrary structures?
Thank you for your attention
Material reconstruction based on Minkowski functionals

Boolean models and Minkowski functionals

Direct relation between Minkowski functionals and permeability?